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Dipole moments of the electron, neutrino and neutron in the MSSM without R-parity symmetry

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Abstract: We show that in the MSSM without R-parity symmetry there are *no* new contributions to electron and neutron electric dipole moments (EDMs) at one-loop induced by the R-parity violating Yukawa couplings. Non-zero EDMs for the electron and neutron first arise at the two-loop level. As an example we estimate the contribution of a two-loop graph which induces electron EDMs. On the other hand, we show that the (Majorana) neutrino electric and magnetic transition moments are non zero even at the one-loop level. Constraints on the R-parity violating couplings are derived from the existing bounds on the neutrino dipole moments.

Keywords: Beyond Standard Model, Supersymmetric Standard Model, Electromagnetic Processes and Properties, CP violation.

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1. Introduction

The electric dipole moment (EDM) d_f, and magnetic dipole moment (MDM) μ_f, of a spin-1/2 particle can be defined by the form factors appearing in the decomposition of the matrix element of the electromagnetic current [1]:

$$\langle f(p')|J_\mu|f(p)\rangle = \bar{u}(p')\Gamma_\mu^{\gamma ff}(q)u(p)\,, \tag{1.1}$$

where

$$\Gamma_\mu^{\gamma ff}(q) = i\,e\left\{\gamma_\mu\left[V_f(q^2) - A_f(q^2)\gamma_5\right] + q^\nu\sigma_{\mu\nu}\left[i\,\frac{\mu_f(q^2)}{e} - \frac{d_f(q^2)}{e}\right]\right\} \tag{1.2}$$

and $q = p' - p$. This formula arises after making use of Lorentz invariance, the Gordon identities and the fact that the external photons and fermions are on shell. The operator which violates CP-symmetry, $\mathcal{L}_{\rm EDM} = -\frac{i}{2}d_f\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}$ is non renormalizable and of dimension five. It reduces to the effective dipole interaction $\mathcal{L}_{\rm EDM} = d_f\vec{\sigma}\cdot\vec{E}$ in the non-relativistic limit.

Experimental searches for electron and neutron EDMs currently provide some of the most severe constraints on new models of particle physics:

$$|d_e| \leq 4.3\times 10^{-27} e\,\mathrm{cm}\ [2]\,, \tag{1.3}$$

$$|d_n| \leq 6.3\times 10^{-26} e\,\mathrm{cm}\ [3]\,. \tag{1.4}$$

All of the contributions to the EDMs or MDMs must be ultraviolate finite because they are non-renormalizable interactions. In addition the interactions flip the chirality of the external fermions and thus break $\mathrm{SU}(2)_L$ invariance. The chirality flip then comes from the fermion masses which in turn come from the spontaneous breakdown of electroweak gauge symmetry. By itself this is able to generate MDMs but not EDMs for which CP-violation is needed. In the SM the required source of CP-violation resides in the complexity of the Yukawa couplings which is parameterized by the CKM-phase.[1] However the CKM-phase has only a tiny contribution of $10^{-30}\,e\,\mathrm{cm}$ to the neutron MDM [1, 5]. The CKM-phase can also penetrate the lepton sector and generate EDMs for the leptons at higher loops but it has been shown that these contributions vanish to three loops [6]. Consequently EDMs are a sensitive test of CP-violation beyond the SM.

If neutrinos are massive (as they indeed appear to be) then CP-symmetry can also be violated in the leptonic sector and one expects neutrino EDMs as well. These EDMs are induced by either a CKM-like phase for Dirac neutrinos or by the three phases for Majorana neutrinos in the leptonic mixing matrix. Since one needs a chirality flip for the EM vertex in order to generate EDMs, Majorana neutrinos cannot have diagonal EDMs or MDMs. They can only have transition electric or magnetic dipole moments [7], i.e. a photon vertex associated with two different neutrino flavours. The experimental bounds on the neutrino dipole moments are divided into two categories: the "Earth bound" constraints

$$|\mu_\nu| \leq 1.5 \times 10^{-10}\ \mu_B\ [8]\,, \tag{1.5}$$

$$|d_{\nu_\tau}| \leq 5.2 \times 10^{-17}\,e\,\mathrm{cm}\ [9]\,, \tag{1.6}$$

and the cosmological ones:

$$|\mu_\nu| \leq 3 \times 10^{-12}\ \mu_B\ [10]\,, \tag{1.7}$$

$$|d_\nu| \leq 2.5 \times 10^{-22}\ e\,\mathrm{cm}\ [11]\,. \tag{1.8}$$

These bounds can be used either for Dirac (diagonal EDMs or MDMs) or for Majorana (transition EDMs or MDMs) neutrinos.

In the SM the EDMs for the leptons due to a possible CP-violation in the leptonic sector are too small to be significant. There is a tendency for the various contributions to cancel or to be proportional to $V_\ell V_\ell^*$. The MDMs of the Dirac neutrinos in the SM with a right-handed singlet were calculated almost twenty years ago [12], and it was found that only a tiny loop induced magnetic moment $\mu_\nu \simeq 3 \times 10^{-19}\mu_B(m_\nu/1\,\mathrm{eV})$ arises. Thus we conclude that in the SM the corrections to the electron, neutron and neutrino EDMs and MDMs are very small and are consistent with the data.

In the MSSM, apart from the CKM-phase or possible CP-violation in the leptonic sector, there are additional sources of CP-violation [13]. The soft breaking masses and couplings can in general be complex, and their phases generate electron and neutron

[1] The main contribution to the EDMs comes from the QCD θ-angle. Here we will assume (for alternatives see the discussion in ref. [4]) a Peccei-Quinn symmetry which is able to set this parameter to zero (albeit at the price of an axion).

EDMs even at the one-loop level in diagrams involving internal squark/sleptons, charginos or gluinos [14]. The current experimental bounds on the EDMs constrain those phases to be less than $\sim 10^{-2}$, unless the phases are flavour off-diagonal, there are cancellations between various contributions or the superpartner masses are of order of 1 TeV [15]. In the case of the neutrino MDM only small corrections have been found in the MSSM with conserved R-parity symmetry [16] and the result turns out to be similar to the case of the SM ($\sim 10^{-19}\mu_B$).

Models that violate R-parity can in principle induce additional contributions to all these parameters [17]. In this paper we examine the effect of breaking R-parity on CP-violating parameters. We extend previous results by presenting a more complete calculation of electron, neutron and neutrino EDM/MDMs in the MSSM without R-parity symmetry.

Before tackling the analysis in detail, we should make clear where our results differ from the previous estimates appearing in the literature. First, we show that any new contributions to the electron and neutron EDMs are small and in fact appear only at two loops. In particular this means that constraints are only on products of 4 or more couplings (either 2 gauge and 2 Yukawa or, as we shall see here, 4 Yukawas). This result is in disagreement with the existing analysis in the literature [18].

Contributions to neutrino EDMs and MDMs can occur at one-loop, however. Babu and Mohapatra [19] were the first to consider the MDM of the neutrino in the context of the MSSM with broken R-parity symmetry. They found contributions of order $\sim 10^{-11}\mu_B$ from the new loop graphs and thus a possible solution to the Solar neutrino problem through the mechanism suggested in ref. [20]. We find results that are smaller than theirs by a factor of 8. Barbieri et al. [21] have also calculated the neutrino MDMs and although we agree numerically with their result to within an order of magnitude, we find that their formula for the neutrino MDMs is unclear. For example, it is not obvious from their analysis that the diagonal neutrino MDM vanishes. Finally, very recently the neutrino MDMs were calculated in ref. [22] in a notation (mass insertion) following closely that of ref. [21]. We agree numerically to within an order of magnitude with these previous estimates of the MDMs. Here we also consider for the first time neutrino EDMs and determine the corresponding bounds.

2. Electron and neutron EDMs

Despite claims in the literature to the contrary [18], the leading contributions to EDMs occur at two loops. In this section we show this using a combination of inspection and power counting arguments, and present a new two-loop diagram which involves only Yukawa couplings.

First consider the extra contributions to the electron EDM from the λ LLE interactions. The EDM is found from the $q \to 0$ limit of d_f in the matrix element of eq. (1.1). Hence the relevant diagrams have one external left-handed, one external right-handed fermion and one photon. Let us denote the number of chiral superfields in the diagram by n_L and n_E, and the number of antichiral superfields by n_{L^*} and n_{E^*}. Adding n_λ of the LLE vertices to a diagram adds $2n_\lambda$ to n_L and n_λ to n_E. In addition we allow D_L and D_E of their respective propagators. Each L propagator removes one L and one L^* and similarly for the E propagators. Finally, we allow D_m propagators with a mass insertion which changes the helicity on a line. Each of these removes one L^* and one E^*. (We also allow D_{m^*} conjugate propagators.) Finally we note that any gauge boson insertion does not change the number of E, E^*, L or L^*.

Insisting that the final diagram has $n_L = 1,\ n_E = 1,\ n_{L^*} = 0,\ n_{E^*} = 0$ yields four equations;

$$\begin{aligned} n_L &= 1 = 2n_\lambda - D_L - D_{m^*}\,, \qquad & n_E &= 1 = n_\lambda - D_E - D_{m^*}\,, \\ n_{L^*} &= 0 = 2n_{\lambda^*} - D_L - D_m\,, \qquad & n_{E^*} &= 0 = n_{\lambda^*} - D_E - D_m\,, \end{aligned} \tag{2.1}$$

giving

$$n_\lambda = n_{\lambda^*}\,, \qquad D_m = D_{m^*} + 1\,, \tag{2.2}$$

i.e. we need at least one mass insertion to flip helicity. Calling the number of non-gauge vertices $V = n_\lambda + n_{\lambda^*}$, the number of non-gauge propagators $I = D_L + D_E + D_m + D_{m^*}$ and the number of non-gauge external legs $E = 2$, we can now use the standard power counting result

$$E = 3V - 2I\,, \qquad L = I - V + 1\,, \tag{2.3}$$

where L is the number of loops. A little more algebra then gives

$$L = n_\lambda\,, \qquad I = 3n_\lambda - 1\,. \tag{2.4}$$

Now consider one-loop diagrams. The above tells us that they must have $n_\lambda = n_{\lambda^*} = 1$ and that they must include at least one mass insertion. Inspection now shows that there are no *irreducible* diagrams of this kind that can give a contribution. Indeed let us consider the contribution from the apparently offending diagram shown in figure 1. The relevant interaction terms come from[2]

$$\begin{aligned} \delta L_L = \frac{1}{2}\lambda_{ijk}\Big\{&\tilde{e}_{L_j}\overline{e}_k P_L \nu_i + \tilde{\nu}_{L_i}\overline{e}_k P_L e_j + \tilde{e}^*_{R_k}\overline{\nu}^c_i P_L e_j - (i \leftrightarrow j)\Big\} + \text{h.c.} + \\ + \lambda'_{ijk}\Big\{&\tilde{d}_{L_j}\overline{d}_k P_L \nu_i + \tilde{\nu}_{L_i}\overline{d}_k P_L d_j + \tilde{d}^*_{R_k}\overline{\nu}^c_i P_L d_j - \\ &- \tilde{u}_{L_j}\overline{d}_k P_L e_i - \tilde{e}_{L_i}\overline{d}_k P_L u_j - \tilde{d}^*_{R_k}\overline{e}_i P_L u_j\Big\} + \text{ h.c.}\,, \end{aligned} \tag{2.5}$$

[2]Here the bilinear term $\kappa_i L_i H_2$ in the superpotential is absent, i.e. κ_i has been set to zero at the electroweak scale.

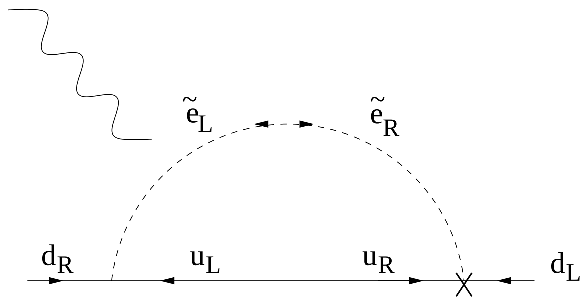

Figure 1: A one-loop diagram for neutron EDMs showing explicitly the required helicity flips. This diagram does not contribute to the EDM in R-parity violating models of supersymmetry because of the absence of the crossed out vertex.

where P_L are projection operators. In general, for lagrangians of the form

$$\phi^* \left(a\overline{\psi}_2 P_R \psi_1 + b\overline{\psi}_2 P_L \psi_1 \right) , \tag{2.6}$$

where ψ and ϕ are generic fermions and scalars, the one-loop contributions to the fermion EDMs are proportional to $\mathrm{Im}(a^* b)$ (see the third reference in [14]). Thus the *same* scalar has to couple to both left and right helicities of a given fermion. For example, in the MSSM the one-loop diagram with an internal chargino gives a contribution to the down EDM because it contains both $\tilde{h}_1$ which couples to $\tilde{u}\, d_R$ *and* $\tilde{h}_2$ which couples to $\tilde{u}\, d_L$ (note the helicity flip required on the up-squark). There is an additional contribution that instead of $\tilde{h}_2$ involves the wino which also couples to $\tilde{u}\, d_L$. On the other hand the gluino gives a one-loop contribution because it is a chargeless particle that can couple to both helicities thanks to its large Majorana mass. Indeed one can check that these three EDM contributions vanish if $\mu H_1 H_2 = 0$, $g_2 = 0$ and $m_{\tilde{g}} = 0$.

Considering the R-parity violating one-loop diagrams with an internal selectron, it is clear that, since there are no interactions that involve u_R, there can be no contribution to the u or d EDMs. (Note that there are extensions of the MSSM that *do* include such an interaction — however these also involve additional multiplets such as isosinglet down quarks coming from the **27** of E_6 [23].) Likewise, λ' only couples the electron to u_L or d_L (and their conjugates) so that there is also no contribution to the electron EDM from the λ' vertex. The diagrams with internal (s)neutrinos can give EDMs only if (like the gluino) the neutrino has a large ($\Delta L = 2$) Majorana mass which of course it does not. Finally we see that the only P_R projection from the λ vertex acts on the neutrino so that this vertex is also unable to contribute to electron or quark EDMs.

We now consider the first Yukawa-only EDM contribution which occurs at two loops and hence must have at least 4 λ or λ' vertices. Examples of the leading diagrams are shown in figure 2, where the additional photon line may be attached

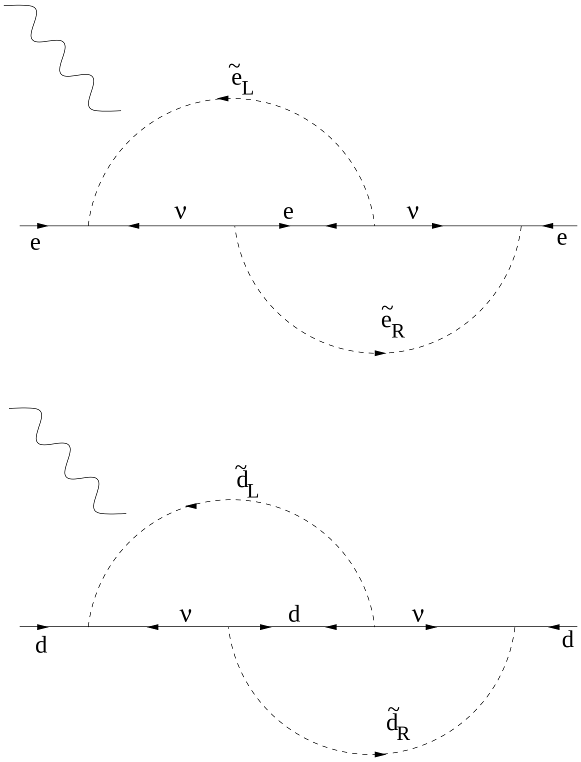

Figure 2: The leading Yukawa-only contributions to electron and neutron EDMs in R-parity require at least two loops and 4 R-parity violating couplings.

to any internal (s)electron or (s)quark. The EDM can be found by extracting the leading linear term in q. For the example where the photon line is attached to the internal electron, we find

$$\begin{aligned}
\Gamma_\mu = e \sum_{ij\ell mn} & m_\ell \, \lambda_{1mn} \, \lambda^*_{j\ell n} \, \lambda^*_{im\ell} \lambda_{ij1} \; \bar{e}^\alpha e_\beta \times \\
& \times \int\!\!\int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2_{\tilde{e}_L}} \frac{1}{p^2 - m^2_{\tilde{e}_R}} \times \\
& \quad \times q_\rho \Bigg\{ 2\gamma^\rho \gamma^\sigma \frac{p_\sigma (k+p)_\mu}{k^2 \left((k+p)^2 - m^2_\ell\right)^2 p^2} + 4\gamma_\mu \gamma^\sigma \frac{p_\sigma (k+p)_\rho}{k^2 \left((k+p)^2 - m^2_\ell\right)^3} + \\
& \qquad + 4\, \gamma^\sigma \gamma^\nu \frac{p^\sigma k_\nu k^\rho (k+p)_\mu}{k^4 ((k+p)^2 - m^2_\ell) p^2} + \\
& \qquad + \gamma^\sigma \gamma_\mu \gamma^\rho \gamma^\nu \frac{k^\sigma p_\nu}{k^2 ((k+p)^2 - m^2_\ell) p^2} \Bigg\}_{\alpha\gamma} P_{L_{\gamma\beta}} \, . \qquad (2.7)
\end{aligned}$$

The F_3 term comes by, for example, writing $\gamma^\mu \gamma^\nu = -i\sigma^{\mu\nu} + g^{\mu\nu}$.

The evaluation of this integral by numerical methods is particularly difficult due to the presence of a kinematical singularity [24, 25]. Instead we note that the full calculation is similar to that of the anomalous magnetic moment of the muon presented in ref. [25]. For the present paper it is therefore sufficient to estimate the resulting EDM as

$$d_e = \sum_{ij\ell mn} \mathrm{Im}\left(\frac{e}{(4\pi)^4 m_{\tilde{e}^2}}(a + b\log z_\ell) m_l\, \lambda_{1mn}\, \lambda^*_{j\ell n}\, \lambda^*_{im\ell}\, \lambda_{ij1}\right), \tag{2.8}$$

where

$$z_\ell = \frac{m_\ell^2}{m_{\tilde{e}^2}} \tag{2.9}$$

and a and b are constants of $\mathcal{O}(1-10)$. Putting in numbers we find that

$$d_e \approx 1.4 \times 10^{-22} e\,\mathrm{cm} \left(\frac{100\,\mathrm{GeV}}{m_{\tilde{e}}}\right)^2 \times$$
$$\times\, \mathrm{Im}\left(\sum_{ij\ell mn} (a + b\log(z_\ell))\, \frac{m_\ell}{m_\tau} \lambda_{1mn}\, \lambda^*_{j\ell n}\, \lambda^*_{im\ell}\, \lambda_{ij1}\right). \tag{2.10}$$

Comparing this number with the experimental constraint of $d_e < 10^{-28}$ [2] we find a bound

$$\mathrm{Im}\left(\sum_{ij\ell mn} \frac{m_\ell}{m_\tau} \lambda_{1mn} \lambda^*_{j\ell n} \lambda^*_{im\ell} \lambda_{ij1}\right) \lesssim 10^{-6}, \tag{2.11}$$

where we conservatively take $a, b = 1$. The equivalent diagram for the neutrons yields

$$d_n \approx 1.4 \times 10^{-20} e\,\mathrm{cm} \left(\frac{100\,\mathrm{GeV}}{m_{\tilde{e}}}\right)^2 \times \mathrm{Im}\left(\sum_{ij\ell mn} \frac{m_\ell}{m_t} \lambda'_{1mn}\, \lambda'^*_{j\ell n}\, \lambda'^*_{im\ell}\, \lambda'_{ij1}\right), \tag{2.12}$$

and comparing with experiment gives

$$\mathrm{Im}\left(\sum_{ij\ell mn} \frac{m_\ell}{m_t} \lambda'_{1mn}\, \lambda'^*_{j\ell n}\, \lambda'^*_{im\ell}\, \lambda'_{ij1}\right) \lesssim 3 \times 10^{-6}. \tag{2.13}$$

Note that in the presence of mixings the above couplings must be rotated into their mass eigenstates. Given the strong bounds already existing on products of couplings [26], it is clear that these constraints from the neutron and electron EDMs are far less important than previously estimated [18]. This is equally the case for the two-loop diagrams with internal gauge bosons discussed in refs. [30, 31].[3]

3. Neutrino MDMs and EDMs

The exception in the discussion of the previous section was the neutrino which *can* get E(M)DM contributions even at one loop. The corresponding diagrams contributing to the neutrino MDM and EDM are shown in figure 3.

[3]Note that these diagrams also satisfy our rules for EDMs. On removing their gauge boson propagators we find a one-loop Yukawa-only diagram, but it is reducible and hence not a contribution to EDMs.

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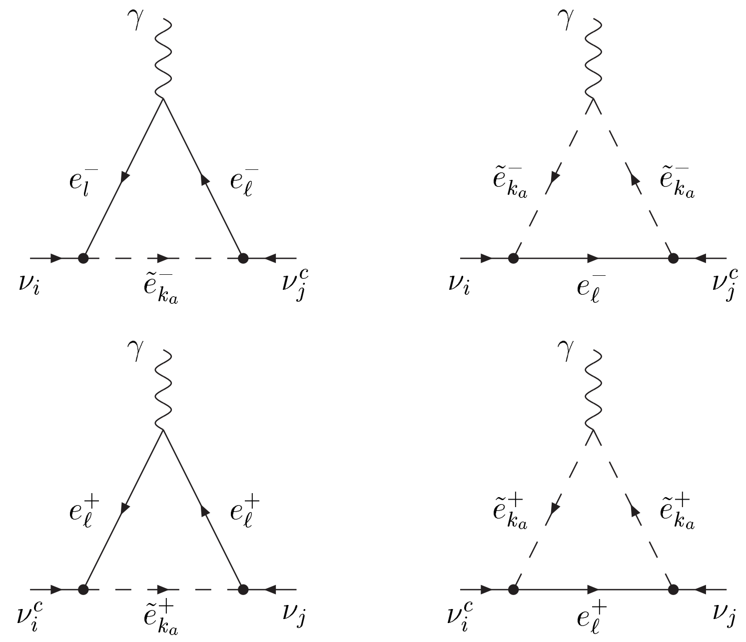

Figure 3: Diagrams contributing to neutrino MDM and EDM from the R-parity violating coupling $\lambda_{ijk}L_iL_j\bar{E}_k$. The equivalent diagrams with the $\lambda'_{ijk}L_iQ_j\bar{D}_k$ vertex are obtained by replacing e with d.

The neutrino MDMs and EDMs in the MSSM without R-parity can be written as:

$$\mu_{ij}^{\nu} = \frac{e}{32\pi^2}\,\mathrm{Re}\left\{\sum_{a=1}^{2} U_{1a}^{\tilde{e}_k} U_{2a}^{*\tilde{e}_k} \sum_{\ell,k=1}^{3} \left(\lambda_{ik\ell}\lambda_{j\ell k} - \lambda_{jk\ell}\lambda_{i\ell k}\right) m_{e_\ell} f\left(m_{e_\ell}^2, m_{\tilde{e}_{k_a}}^2\right) + \right. \tag{3.1}$$

$$\left. + \sum_{a=1}^{2} U_{1a}^{\tilde{d}_k} U_{2a}^{*\tilde{d}_k} \sum_{\ell,k=1}^{3} \left(\lambda'_{ik\ell}\lambda'_{j\ell k} - \lambda'_{jk\ell}\lambda'_{i\ell k}\right) m_{d_\ell} f\left(m_{d_\ell}^2, m_{\tilde{d}_{k_a}}^2\right)\right\},$$

$$d_{ij}^{\nu} = -\frac{e}{32\pi^2}\,\mathrm{Im}\left\{\sum_{a=1}^{2} U_{1a}^{\tilde{e}_k} U_{2a}^{*\tilde{e}_k} \sum_{\ell,k=1}^{3} \left(\lambda_{ik\ell}\lambda_{j\ell k} - \lambda_{jk\ell}\lambda_{i\ell k}\right) m_{e_\ell} f\left(m_{e_\ell}^2, m_{\tilde{e}_{k_a}}^2\right) + \right. \tag{3.2}$$

$$\left. + \sum_{a=1}^{2} U_{1a}^{\tilde{d}_k} U_{2a}^{*\tilde{d}_k} \sum_{\ell,k=1}^{3} \left(\lambda'_{ik\ell}\lambda'_{j\ell k} - \lambda'_{jk\ell}\lambda'_{i\ell k}\right) m_{d_\ell} f\left(m_{d_\ell}^2, m_{\tilde{d}_{k_a}}^2\right)\right\},$$

where the function $f(x,y)$ is[4]

$$f(x,y) = \frac{1}{y}\left(2 + \ln\frac{y}{x}\right). \tag{3.3}$$

The matrix $U^{\tilde{e}}(U^{\tilde{d}})$ diagonalizes the slepton (down squark) mass matrix and is given[5]

[4]We have made use of the approximation $m_\ell \ll m_{\tilde{l}}$ and $m_q \ll m_{\tilde{q}}$.

[5]Here we assume that the soft SUSY CP-phases are small. Additional contributions to the EDMs for the neutrinos are possible if they are large.

in terms of the mixing angle $\theta_{\tilde{e}_i}(\theta_{\tilde{d}_i})$

$$U^{\tilde{e}_i/\tilde{d}_i} = \begin{pmatrix} \cos\theta_{\tilde{e}_i/\tilde{d}_i} & -\sin\theta_{\tilde{e}_i/\tilde{d}_i} \\ \sin\theta_{\tilde{e}_i/\tilde{d}_i} & \cos\theta_{\tilde{e}_i/\tilde{d}_i} \end{pmatrix}. \tag{3.4}$$

The largest MDMs and EDMs are obtained when the product $U^{\tilde{f}_k}_{1a} U^{*\tilde{f}_k}_{2a}$ is maximised. This is achieved for maximal mixing: $\sin 2\theta_{\tilde{e}_i/\tilde{d}_i} = 1$ and we have $U^{\tilde{f}_k}_{1a} U^{*\tilde{f}_k}_{2a} = 1/2$. The off-diagonal elements of the scalar fermion mass matrix are proportional to the fermion Yukawa coupling and thus typically small. We thus do not expect to be close to maximal mixing. However, large mixing can be obtained when the SUSY soft breaking masses of the slepton (squark) doublets, $m_{\tilde{L}_i}(m_{\tilde{Q}_i})$, are equal to those of the right-handed singlets, $m_{\tilde{e}_i}(m_{\tilde{d}_i}, m_{\tilde{U}_i})$, and the tri-linear coupling $A = \mathcal{O}(500\,\mathrm{GeV})$ is large. As we wish to show below that the bounds on the R_p-violating couplings from the MDMs and EDMs are typically weak, we continue with the assumption of maximal mixing and universal scalar fermion masses. We will then see that in this most optimistic case the bounds are indeed weak and can thus conclude that this is true generally. By expanding the sum over the mass eigenstates of the sleptons (squarks) we obtain,

$$\begin{aligned}\mu^\nu_{ij} = \frac{e}{64\pi^2}\,\mathrm{Re}\Bigg\{ &\sum_{\ell,k=1}^{3} (\lambda_{ik\ell}\lambda_{j\ell k} - \lambda_{jk\ell}\lambda_{i\ell k})\, m_{e_\ell}\Big[f\Big(m^2_{e_\ell}, m^2_{\tilde{e}_{k_1}}\Big) - f\Big(m^2_{e_\ell}, m^2_{\tilde{e}_{k_2}}\Big)\Big] + \\ &+ \sum_{\ell,k=1}^{3} (\lambda'_{ik\ell}\lambda'_{j\ell k} - \lambda'_{jk\ell}\lambda'_{i\ell k})\, m_{d_\ell}\Big[f\Big(m^2_{d_\ell}, m^2_{\tilde{d}_{k_1}}\Big) - f\Big(m^2_{d_\ell}, m^2_{\tilde{d}_{k_2}}\Big)\Big]\Bigg\}, \end{aligned} \tag{3.5}$$

$$\begin{aligned}d^\nu_{ij} = -\frac{e}{64\pi^2}\,\mathrm{Im}\Bigg\{ &\sum_{\ell,k=1}^{3} (\lambda_{ik\ell}\lambda_{j\ell k} - \lambda_{jk\ell}\lambda_{i\ell k})\, m_{e_\ell}\Big[f\Big(m^2_{e_\ell}, m^2_{\tilde{e}_{k_1}}\Big) - f\Big(m^2_{e_\ell}, m^2_{\tilde{e}_{k_2}}\Big)\Big] + \\ &+ \sum_{\ell,k=1}^{3} (\lambda'_{ik\ell}\lambda'_{j\ell k} - \lambda'_{jk\ell}\lambda'_{i\ell k})\, m_{d_\ell}\Big[f\Big(m^2_{d_\ell}, m^2_{\tilde{d}_{k_1}}\Big) - f\Big(m^2_{d_\ell}, m^2_{\tilde{d}_{k_2}}\Big)\Big]\Bigg\}. \end{aligned} \tag{3.6}$$

Note that the diagrams of the first row in figure 3 differ by a sign from those of the second row (due to the photon vertex) and an interchange of the indices $i \leftrightarrow j$ (see eqs. (3.1) and (3.2)).

Some remarks are in order here:

- The diagonal elements of the MDMs and EDMs of the neutrinos are zero, i.e. $\mu^\nu_{ij} = d^\nu_{ij} = 0$. This is of course a general statement for the Majorana neutrinos. $i \neq j$ is assumed below.

- For $k = \ell$ the contribution to eqs. (3.5) and (3.6) for both the neutrino MDMs and EDMs are zero.

- If one R-parity violating coupling dominates over the others then again their contributions to the neutrino MDMs and EDMs are zero. It is known [26] that even if we assume one coupling at a time at the GUT scale a number of lepton number violating couplings appear at the electroweak scale since there is no symmetry (lepton symmetry) to protect them. However, we find that the effect on the neutrino MDMs and EDMs is tiny [27].

- If the sleptons and the squarks mass eigenstates are nearly degenerate then the MDM and EDM for the neutrinos are much less than the experimental constraints.

In passing we note that the CKM-phase does not enter into the one-loop expression for the neutrino EDM. The effect is thus purely due to the R-parity violating couplings.

We now consider the contributions to neutrino MDMs. The importance of each term in eq. (3.5) depends on which are the dominant R-parity violating couplings and also on the degeneracy of the slepton and squark mass eigenstates. Here we shall assess the maximum contribution of the RPV couplings to the neutrino MDMs by taking one of the two slepton/squark mass eigenstates e.g $m_{\tilde{e}_2}, m_{\tilde{d}_2}$ to be in the decoupling region (the function $f(x,y)$ goes to zero for large y). Beacom and Vogel [8] have recently shown that for Majorana neutrinos with two flavours the neutrino MDMs are given by

$$\mu^2_{\nu_e} = |\mu_{12}|^2\,, \tag{3.7}$$

for either vacuum or MSW mixing, and the bound obtained from SuperKamiokande solar neutrino data [29] is

$$|\mu_{\nu_e}| \leq 1.5 \times 10^{-10} \mu_B \;\; (90\%\, CL)\,. \tag{3.8}$$

By using this bound and assuming that the sleptons and squarks of each generation are almost degenerate, i.e. $m_{\tilde{e}} = m_{\tilde{\mu}} = m_{\tilde{\tau}}$ and $m_{\tilde{d}} = m_{\tilde{s}} = m_{\tilde{b}}$ we find

$$\begin{aligned}
&\lambda_{121}\lambda_{212}\Big(m_e f\left(m_e^2, m_{\tilde{e}}^2\right) - m_\mu f\left(m_\mu^2, m_{\tilde{e}}^2\right)\Big) + \\
&\quad +\lambda_{131}\lambda_{213}\Big(m_e f\left(m_e^2, m_{\tilde{e}}^2\right) - m_\tau f\left(m_\tau^2, m_{\tilde{e}}^2\right)\Big) + \\
&\quad +\lambda_{123}\lambda_{232}\Big(m_\tau f\left(m_\tau^2, m_{\tilde{e}}^2\right) - m_\mu f\left(m_\mu^2, m_{\tilde{e}}^2\right)\Big) + \\
&\quad +\Big(\lambda'_{121}\lambda'_{212} - \lambda'_{221}\lambda'_{112}\Big)\Big(m_d f\left(m_d^2, m_{\tilde{d}}^2\right) - m_s f\left(m_s^2, m_{\tilde{d}}^2\right)\Big) + \\
&\quad +\Big(\lambda'_{131}\lambda'_{213} - \lambda'_{231}\lambda'_{113}\Big)\Big(m_d f\left(m_d^2, m_{\tilde{d}}^2\right) - m_b f\left(m_b^2, m_{\tilde{d}}^2\right)\Big) + \\
&\quad +\Big(\lambda'_{132}\lambda'_{223} - \lambda'_{123}\lambda'_{232}\Big)\Big(m_s f\left(m_s^2, m_{\tilde{d}}^2\right) - m_b f\left(m_b^2, m_{\tilde{d}}^2\right)\Big) \leq 10^{-4}\,.
\end{aligned} \tag{3.9}$$

For $m_{\tilde{e}} = m_{\tilde{d}} = 100\,\mathrm{GeV}$ and one dominant pair of R-parity violating couplings at a time we obtain the following bounds:

$$\mathrm{Re}(\lambda_{121}\lambda_{212}) < 0.58\,, \tag{3.10}$$

$$\mathrm{Re}(\lambda_{131}\lambda_{213}) < 0.059\,, \tag{3.11}$$

$$\mathrm{Re}(\lambda_{123}\lambda_{232}) < 0.063\,, \tag{3.12}$$

$$\mathrm{Re}(\lambda'_{121}\lambda'_{212})\,,\ \mathrm{Re}(\lambda'_{221}\lambda'_{112}) < 0.60\,, \tag{3.13}$$

$$\mathrm{Re}(\lambda'_{131}\lambda'_{213})\,,\ \mathrm{Re}(\lambda'_{231}\lambda'_{113})\,,\ \mathrm{Re}(\lambda'_{132}\lambda'_{223})\,,\ \mathrm{Re}(\lambda'_{123}\lambda'_{232}) < 0.030\,. \tag{3.14}$$

We recall that these bounds were obtained for the optimistic case of maximal mixing. For smaller mixing they are correspondingly weaker. If we now compare these bounds with those shown in ref. [26], we find that they are all far more relaxed than the constraints obtained from other processes (even in some cases more relaxed than the individual bounds on the corresponding RPV couplings). Thus we conclude that the contribution from the R-parity violating couplings to the neutrino MDMs is rather small.

It is possible in general to start with complex RPV couplings. Assuming no neutrino mixing here the induced neutrino EDMs[6] are given by [10]

$$d^{\nu} = \frac{1}{2}\sum_{i,j=1}^{3} |d_{ij}|^2 = |d_{12}|^2 + |d_{13}|^2 + |d_{23}|^2\,. \tag{3.15}$$

By considering the cosmological bound[7] of [11] $d^{\nu} = 2.5 \times 10^{-22}\,e\,\mathrm{cm}$ and assuming one dominant pair of RPV couplings at a time we obtain ($m_{\tilde{e}} = m_{\tilde{d}} = 100\,\mathrm{GeV}$):

$$\mathrm{Im}(\lambda_{i21}\lambda_{j12}) < 0.05\,, \tag{3.16}$$

$$\mathrm{Im}(\lambda_{i31}\lambda_{j13}) < 0.004\,, \tag{3.17}$$

$$\mathrm{Im}(\lambda_{i23}\lambda_{j32}) < 0.005\,, \tag{3.18}$$

$$\mathrm{Im}(\lambda'_{i21}\lambda'_{j12}) < 0.06\,, \tag{3.19}$$

$$\mathrm{Im}(\lambda'_{i31}\lambda'_{j13})\,,\ \mathrm{Im}(\lambda'_{i32}\lambda'_{j23}) < 0.0024\,. \tag{3.20}$$

These new EDM results conclude this chapter and the discussion on neutrino MDMs and EDMs.

[6] There are no neutrino MDMs in this case.

[7] The accelerator bound of [9] is almost five orders of magnitude more relaxed than the cosmological one and does not constrain the RPV couplings at all.

4. Conclusions

We have shown that in the MSSM without R-parity symmetry it is impossible to generate additional electron and neutron EDMs at one-loop from the R-parity violating Yukawa couplings. EDMs for the electron and neutron first arise at the two-loop level and we estimated the contribution of the new two-loop graphs. We found that Yukawa-only diagrams can give constraints on products of 4 R-parity violating Yukawa couplings;

$$\begin{aligned} \mathrm{Im}\left(\sum_{ij\ell mn} \frac{m_\ell}{m_\tau} \lambda_{1mn}\lambda^*_{j\ell n}\lambda^*_{im\ell}\lambda_{ij1}\right) &\lesssim 10^{-6}\,, \\ \mathrm{Im}\left(\sum_{ij\ell mn} \frac{m_\ell}{m_t} \lambda_{1mn}\lambda^*_{j\ell n}\lambda^*_{im\ell}\lambda_{ij1}\right) &\lesssim 3\times 10^{-6}\,. \end{aligned} \tag{4.1}$$

Conversely we find that (Majorana) neutrino electric and magnetic transition moments are non zero even at the one-loop level. Upper bounds on the R-parity violating couplings were derived using the current experimental constraints on neutrino dipole moments.

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Note added in proof: Whilst in the final stages of preparation ref. [30] appeared which draws the same conclusion concerning the electron and neutron EDMs. Also, stringent bounds on the imaginary parts of the products of the R-parity violating couplings from P,T-violating electron-nucleon interactions in the R-parity violating MSSM appeared in a very resent paper, [31].

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